BANK PROBATIONARY OFFICER
QUANTITATIVE APTITUDE

PROBABILITY

If an event can happen in 'a' ways and fail in 'b' ways, and each of these ways is equally likely, then probability or the chance, or its happening is $\frac{a}{a+b}$, and that of its failing is $\frac{b}{a+b}$.

e.g., If in a lottery there are 7 prizes and 25 blanks, the chance that a person holding 1 ticket will win a prize is $\frac{7}{32}$, and his chance of not winning is $\frac{25}{32}$.

If p is the probability of the happening of an event, the probability of its not happening is $1-p$.

Instead of saying that the chance of the happening of an event is $\frac{a}{a+b}$, it is sometimes stated that the odds are 'a' to 'b' in favour of the event, or 'b' to 'a' against the event.

If c is the total no. of cases, each being equally likely to occur, and of these 'a' are favourable to the event, then the probability that the event will happen is $\frac{a}{c}$, and the probability that it will not happen is $1-\frac{a}{c}$.

ex.1. What is the chance of throwing a number greater than 4 with an ordinary die whose faces are numbered from 1 to 6.

Sol: There are 6 possible ways in which the die can fall, and of these two are favourable to the event required.

$\therefore$ required chance $=\frac{2}{6}=\frac{1}{3}$.

ex.2. From a bag containing 4 white and 5 black balls a man draws 3 at random, what are the odds against these being all black?

Sol: The total no. of ways in which 3 balls can be drawn is ${}^9C_3$ and the no. of ways of drawing 3 black balls is ${}^5C_3$; therefore the chance of drawing 3 black balls is

$$\frac{{}^5C_3}{{}^9C_3}=\frac{5.4.3}{9.8.7}=\frac{5}{42}$$

Thus the odds against the event are 37 to 5.

ex.3. Find the chance of throwing at least one ace in a simple throw with two dice.

Sol: The possible no. of cases is 6×6 or 36.

An ace on one die may be associated with any of the 6 numbers on the other die, and the remaining 5 numbers on the first die may be associated with the ace on the second die, thus the number of favourable cases is 11.

$\therefore$ Required chance $=\frac{11}{36}$

ex.4. Find the chance of throwing more than 15 in one throw with 3 dice.

Sol: A throw amounting to 18 must be made up of 6, 6, 6 and this can occur in 1 way, 17 can be made up of 6, 6, 5 which can occur in 3 ways, 16 may be made up of 6, 6, 4 and 6, 5, 5 each of which arrangements can occur in 3 ways.

Therefore the no.of favourable cases is = 1+ 3 + 3 + 3 = 10 and the total number of cases is $6^3=216$ $\therefore$ required chance

$$=\frac{10}{216}=\frac{5}{108}$$

ex.5. What is the probability that a digit selected at random from the logarithmic table is (i) 1, (ii) 3 or 7.

Sol: Various digits in the log table are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, i.e. a total of 10 digits is used.

(i) The number of favourable cases for getting 1, out of 10 all equally likely cases is one $\therefore$ Prob. of getting 1 is $\frac{1}{10}$.

(ii) The no. of favourable cases for getting 3 or 7 is 2. $\therefore$ Required Prob. $= \frac{2}{10} = \frac{1}{5}$.

ex. 6. Find out the probability of forming 563 of 169 with the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 when only numbers of three digits are formed and when

(i) repetitions are not allowed.

(ii) repetitions are allowed.

Sol: (i) **when repetitions are not allowed:** The total ways of forming numbers of three digits with 9 given digits is ${}^9P_3 = 9 \times 8 \times 7 = 504$. Of these, favourable are two, i.e. 563 and 169. So the required probability is $\frac{2}{504} = \frac{1}{252}$

(ii) When repetitions are allowed: Out of given 9 digits, 3 digit numbers can be formed in $9 \times 9 \times 9 = 729$ ways. Of these two are favourable cases,

Probability $= \frac{2}{729}$

ex.7. In a race where 12 horses are running, the chance that horse A will win is $\frac{1}{6}$, that B will win is $\frac{1}{10}$ and that C will win is $\frac{1}{8}$. Assuming that a dead heat is impossible, find the chance that one of them will win.

Sol: Probability that A wins $(p_1) = \frac{1}{6}$, that B wins $(p_2) = \frac{1}{10}$ and that C wins $(p_3) = \frac{1}{8}$. As a dead heat is impossible, these are mutually exclusive events, so the chance that one of them will win the race is $p_1 + p_2 + p_3$ i.e. $\frac{1}{6} + \frac{1}{10} + \frac{1}{8} = \frac{47}{120}$.

ex.8. Two balls are to be drawn from a bag containing 5 red and 7 white balls; find the chance that they will both be white.

Sol: Here any one pair of balls is as likely to be drawn as any other pair. The total number of pairs is ${}^{12}C_2$, and the number of pairs which are both white is ${}^7C_2$, .

The required chance is therefore $\frac{7.6}{1.2} \quad \frac{12.11}{1.2} = \frac{7}{22}$.

ex.9. Find the chance of drawing 2 white balls in succession from a bag conatining 5 red and 7 white balls, the balls drawn not being replaced.

Sol: The chance of drawing a white ball the first time is $\frac{7}{12}$; and, having drawn a white ball the first time, there will be 5 red and 6 white balls left, and therefore the chance of drawing a white ball the second time will be $\frac{6}{11}$.

Hence the chance of drawing two white balls in succession will be $\frac{7}{12} \times \frac{6}{11} = \frac{7}{22}$.

ex. 10. There are two bags, one of which contains 5 red and 7 white balls and the other 3 red and 12 white balls, and a ball is to be drawn from one or other of the two bags; find the chance of drawing a red ball.

Sol: The chance of choosing the first bag is $\frac{1}{2}$, and if the first bag be chosen the chance of drawing a red ball from it is $\frac{5}{12}$; hence the chance of drawing a red ball from the first bag is $\frac{1}{2} \times \frac{5}{12} = \frac{5}{24}$. Similarly the chance of drawing a red ball from the second bag is $\frac{1}{2} \times \frac{3}{15} = \frac{1}{10}$. Hence as these events are mutually exclusive, the chance required is $\frac{5}{24} + \frac{1}{10} = \frac{37}{120}$.

ex. 11. In two bags there are to be put altogether 2 red and 10 white balls, neither bag being empty. How must the balls be divided so as to give a person who draws one ball from either bag.

Sol: (i) the least chance and (ii) the greatest chance of drawing a red ball.

(i) The least chance is when one bag contains only one white ball, and the greatest chance is when one bag contains only one red ball, the chance being $\frac{1}{11}$ and $\frac{6}{11}$ respectively.

EXERCISE

1. From a pack of 52 cards, two are drawn at random. Find the chance that one is a knave and the other a queen.

   a) $6/663$ b) $8/663$

   c) $663/8$ d) $52/663$

   e) None

2. A bag contains 5 white, 7 black, and 4 red balls. Find the chance that three balls drawn at random are all white.

   a) $1/56$ b) $2/56$

   c) $3/56$ d) $4/56$

   e) None of these

3. What is the chance of throwing an ace in only the first of two successive throws with an ordinary dice.

   a) $3/36$ b) $4/36$

   c) $5/36$ d) $7/36$

   e) None of these

4. Three cards are drawn at random from an ordinary pack; find the chance that they will consist of a knave, a queen and a king.

   a) $\frac{11}{5525}$ b) $\frac{12}{5525}$

   c) $\frac{15}{5525}$ d) $\frac{16}{5525}$

   e) None of these

5. If 8 coins are tossed, what is the chance that one and only one will turn up head?

   a) $1/32$ b) $3/32$

   c) $5/32$ d) $7/32$

   e) None of these

6. In a certain town, the ratio of males to females is 1000 : 1987. If this tendency is expected to continue, what is the chance that a newly born baby is male?

a) $\frac{1000}{1987}$ b) $\frac{1000}{2987}$

c) $\frac{1987}{1000}$ d) $\frac{2987}{1000}$

e) None

7. What is the chance that a leap year, selected at random, will contian 53 sundays?

a) $\frac{1}{7}$ b) $\frac{2}{7}$

c) $\frac{3}{7}$ d) $\frac{4}{7}$

e) None

8. Out of all the integers from 1 to 100, a number is selected at random. what is the probability that the selected number is not divisible by 7 ?

a) $\frac{40}{50}$ b) $\frac{41}{50}$

c) $\frac{42}{50}$ d) $\frac{43}{50}$

e) None of these

9. 100 students appeared for two examinations. 60 passed the first, 50 passed the second and 30 passed both. Find the probability that a student selected at random has failed in both the examinations.

a) $\frac{1}{5}$ b) $\frac{2}{5}$

c) $\frac{3}{5}$ d) $\frac{4}{5}$

e) None

10. A child is asked to pick up two balloons from a box containing 10 blue and 15 red balloons. What is the probability of the child picking at random two balloons of different colours ?

a) $\frac{1}{2}$ b) $\frac{1}{3}$

c) $\frac{1}{4}$ d) $\frac{1}{5}$

Answers:

1. (b)	2. (a)	3. (c)	4. (d)
5. (a)	6. (a)	7. (b)	8. (d)
9. (a)	10. (a)		

INEQUALITIES AND MAXIMA & MINIMA

Any quantity is said to be greater than another quantity b when a - b is positive. Thus, 2 is greater than -3 as 2 - (-3) = 5 is +ve.

Also, b is said to be less than a when b-a is -ve. Thus - 5 less than - 2, because - 5 - (-2)= -3, which is -ve.

Zero must be regarded as greater than any -ve quantity and less than any +ve quantity.

RULE 1- An inequality will still hold after each side has been increased, diminished, multiplied or divided by the same +ve quantity.

i.e., if a > b,

a + c > b + c

a - c > b - c

ac > bc c is positive

$$\frac{a}{c} > \frac{b}{c}$$

RULE 2 - In an inequality any term may be transposed from one side to the other if its sign is changed. i.e. if a - c> b, then a> b+c, or -c>b-a.

RULE 3 - If the sides of all the terms of an inequality be changed, the sign of the inequality must be reversed. i.e. if a>b, then b < a.

RULE 4 - If the signs of all the terms of an inequality be changed, the sign of the inequality must be reversed. ie. if a> b, then -a< -b or -ac < -bc, where c is +ve.

RULE 5 - If the sides of an inequality be mutiplied by the same -ve quantity, the sign of the inequality must be reversed. i.e. if a > b, then -ac < -bc; where c is +ve.

RULE 6- If a>b, then $a^n > b^n$, and

$1/a^n < 1/b^n$ or $a^{-n} < b^{-n}$; if n is a +ve quantity.

RULE 7 - The square of every real quantity is +ve and therefore must be greater than zero.

i.e. $(a-b)^2 > 0;\ a^2 + b^2 > zab;$ Similarly,

$\frac{x+y}{2} > \sqrt{xy}, x > 0,\ y > 0$

Hence the arithmetic mean of two +ve quantities is greater than their geometric mean.

RULE 8 - If the sum of two +ve quantities is given, their product is greatest when they are equal; and if the product of two +ve quantities is given, their sum is least when they are equal.

RULE 9 - If a, b, c, ...k are n unequal quantities, then,

$$\left(\frac{a+b+c+......+k}{n}\right)^n > a.b.c.d.......k$$

i.e. $\frac{a+b+c+.........+k}{n} > (a.\ b.\ c.\ .....k)$

Therefore , the arithmetic mean of any number of +ve quantities is greater than their geometric mean.

RULE 10- If a and b are positive and unequal,

$\frac{a^m + b^m}{2} > \left(\frac{a+b}{2}\right)^m$, except when 'm' is a position proper fraction.

If m is a positive integer or any negative quantity $\frac{a^m b^m}{2} > \left(\frac{a+b}{2}\right)^m$

If m is positive and less than1,

$$\frac{a^m + b^m}{2} < \left(\frac{a+b}{2}\right)^m$$

If there are n positive quantities a, b, c, ...k, then

$$\left(\frac{a^m + b^m + c^m + ...k^m}{n}\right) > \left(\frac{a+b+c+.....+k}{n}\right)^m$$

unless m is a positive proper fraction.

RULE 11- It a, b, c are +ve and not all equal, then (a + b + c) (ab+ bc + ca) > 9 abc and, (b + c) (c + a) (a + b) > 8 abc.

RULE 12- $\frac{a+x}{b+x} > OR < \frac{a}{b}$ according as a < OR > b is x be positive or according as a> OR <b if x is negative.

RULE 13- $\frac{a+c+e+......}{b+d+f+......}$ is less than the greatest and greater than the least, of the fractions $\frac{a}{b}, \frac{c}{d}, \frac{e}{f}$,.......

RULE 14- If a>x , b>y, c>z then a + b + c + ...> x + y + z +.... and abc......> x y z........

RULE 15- $a^2 + b^2 + c^2 \geq bc + ca + ab$.

RULE 16 - $(\lfloor n)^2 > n^n$.

RULE 17- For any positive integer n

$$2 \leq (1+\frac{1}{n})^n \leq 3$$

RULE 18 - $a^2 b + b^2 c + c^2 a \geq 3abc$

RULE 19 - $\frac{a}{b} + \frac{b}{c} + \frac{c}{d} + \frac{d}{a} > 4$

RULE 20 - $a^4 + b^4 + c^4 + d^4 \geq 4abcd$

ex.1. Which of the two number $(1.000001)^{1000000}$ and 2 is greater?

Sol: $(1.000001)^{1000000} = (1+\frac{1}{1000000})^{1000000}$
Which is greater than 2. (RULE 17).

ex. 2. which of the two numbers 1000^{1000} and 1001^{999} is greater ?

Sol: $\frac{1001^{999}}{1000^{1000}} = \left(\frac{1001}{1000}\right)^{1000} \cdot \frac{1}{1001} =$

$\left(1+\frac{1}{1000}\right)^{1000} \cdot \frac{1}{1001}$

which is $<3 \cdot \frac{1}{1001} < 1$

$\therefore 1000^{1000} > 1001^{999}$

ex. 3. Find that min value of $x^2 - 4x + 7$ for real values of x.
$x^2 - 4x + 7 = (x-2)^2 + 3$

Sol: A perfect square is always positive, i.e., it cannot be less than zero. $\therefore$ the given expression is least when $(x-2)^2 = 0$

$\therefore$ min: value = 3

ex . 4 Find the maximum value of $3.5 + 4x - 4x^2$ for real values of x.

Sol: $3.5 + 4x - 4x^2 = 4.5 - (1 - 4x + 4x^2)$

$= 4.5 - (1-2x)^2$

$\therefore$ the given expression is maximum when $(1-2x)^2$ is least. ie, when $1-2x = 0$.

$\therefore$ maximum value = 4.5 and this occurs when $x = \frac{1}{2}$.

ex. 5. Solve $2x + 4 < \frac{20}{3}$

Sol: $2x < \frac{20}{3} - 4 \therefore 2x < \frac{8}{3} \therefore x < \frac{4}{3}$

ex. 6. Solve (-2x + 3) $\leq$ 6
$-2x \leq 3 \therefore x \geq -\frac{3}{2}$

ex. 7. Solve $(x-3)\ (x+4) > 0$
$(x-3)(x-(-4) > 0;$

$\therefore x$ does not lie between - 4 and 3.

ex. 8. Solve $x^2 + 8x + 7 < 0$
$(x+1)\ (x+7) < 0;$

$= [\,x-(-1)]\,[x-(-7)] < 0$

$\therefore x$ lies between - 7 and -1.

ex. 9 Slove $x^2 - 8x + 7 > 0$
$(x-7)(x-1) > 0;$ $\therefore x$ does not lie between 1 and 7.

ex. 10. If w satisfies both the following in equalities, and w is an integer, what values can w have ?

(i) $5(w+10) - 4w > 0$

(ii) $8 + 7w < 3(2w+1)$

Sol: $5w + 50 - 4w > 0; \therefore w > -50$

$8 + 7w < 6w + 3; \therefore w < -5$ $\therefore w$ lies between -50 and -5.

ex. 11. $P = \left[\frac{x}{3} + 2 > 2\,\frac{2}{3}\right];$

$Q = \left[12 - 2x > 8 - x\right]$

$R = \left[10 - 4x < 14\right];$

$S = \left[0, 1, 2, 3, 4, 5\right]$

If x is an integer, list the members of the following sets;

(a) $P \cap S$ (b) $Q \cap S$

(c) $R \cap S$ (d) $P \cap Q \cap S$

(e) $(P \cup R) \cap S$

Sol: P : $\frac{x}{3} + 2 > 2\frac{2}{3}$ i.e. $\frac{x}{3} > \frac{2}{3}$

$\therefore P = [3, 4, 5.....]$

Q : $12 - 2x > 8 - x \; i.e. -x > -4; \; x < 4,$

$\therefore Q = [3, 2, 1, 0, -1, ...]$

$R = 10 - 4x < 14 \; i.e. -4x < 4, \; x < 1,$

$\therefore R = [0, 1, 2, 3, ...]$

Therefore, $P \cap S = [3, 4, 5]$

$Q \cap S = [0, 1, 2, 3]$

$R \cap S = [0, 1, 2, 3, 4, 5]$

$P \cap Q \cap S = [3]$

$(P \cap R) \cap S = [0, 1, 2, 3, 4, 5]$

(a) 3, 4, 5 (b) 0,1,2,3

(c) 0,1,2,3,4,5 (d) 3

(e) 0, 1, 2, 3, 4, 5.

ex. 12. Between what values of x, is the expression $19x - 2x^2 - 35$ positive?

Sol: Let y denote the given expression,

$y = -(2x^2 - 19x + 35)$

$= -(2x - 5)(x - 7)$

$= (2x - 5)(7 - x) = 2(x - \frac{5}{2})(7 - x)$

when $x < \frac{5}{2}; x - \frac{5}{2}$ is -ve and (7 - x) is +ve,

$\therefore$ y is negative

When $x > \frac{5}{2} \; but < 7; \; x - \frac{5}{2}$ +ve and $7 - x$ is $+ve$. $\therefore y$ is positive.

when $x > 7, \; x - \frac{5}{2}$ is +ve and 7-x is -ve; $\therefore$ y is -ve.

$\therefore$ The given expression is positive only as long as x is between $2\frac{1}{2}$ and 7.

ex. 13. Find the greatest value of $(a + x)^3 \; (a - x)^4$ for any real value of x.

The given expresssion is greatest when:

$\frac{a + x}{3} = \frac{a - x}{4}$ OR $x = -\frac{a}{7}$.

Thus the greatest value is $\frac{6^3 \cdot 8^4}{7^7} a^7$.

[Note : $a^m b^n c^p$........... will be greatest when the factors $\frac{a}{m} = \frac{b}{n} = \frac{c}{p} = ...$]

Remember : In order that $ax^2 + bx - c$ may be always +ve, $b^2 - 4ac$ must be -ve or 0, and a must be +ve. In order that $ax^2 + bx + c$ may be always negative, $b^2 - 4ac$ must be -ve or 0 and a must be -ve.

ex. 14 Find the greatest value of $\frac{x + 2}{2x^2 + 3x + 6}$ for real values of x.

Sol: Let $Y = \frac{x + 2}{2x^2 + 3x + 6}$; then

$2yx^2 + (3y-1)\,x + 6y - 2 = 0$.

If x is real, $(3y-1)^2 - 8y(6y-2)$ must be positive $\therefore$ (1+13y) $(1-3y)$ must be +ve. Hence y must lie between $\frac{1}{3}$ and $-\frac{1}{13}$ and its greatest value is $\frac{1}{3}$.

ex. 15: Find the least value of $3x + 4y$ if $x^2\ y^3 = 6,$ x and y are positive.

Sol: Let $A = \frac{3}{2}x$; $B = \frac{4}{3}y$;

$$\therefore x^2 y^3 = \frac{4A^2}{9} \cdot \frac{27}{64}B^3 = 6$$

$$\therefore A^2\ B^3 = 32$$

Now, 3x + 4y = 2A + 3 B. also, A.A.B.B.B = 32. The least value of A+A+B+B+B will be so when all the quantities are equal. (RULE 8).

$\therefore A = B = 2$; $\therefore$ Least value of $3x + 4y = 2\times 2 + 3\times 2 = 10$.

ex. 16. If x may have any real value, find which is greater, $x^3 + 16x$ OR $7x^2 + 10$.

Sol: $x^3 + 16x - 7x^2 - 10$ has a factor x-1.

$$\therefore x^3 - 7x^2 + 16x - 10$$
$$= (x-1)(x^2 - 6x + 10)$$
$$= (x-1)[(x-3)^2 + 1]$$

The Second factor is always positive, hence $x^3 + 16x$ is greate if $x > 1$.

EXERCISE

1. If x be positive, find the greatest values of $(5-x)(x+3)$

   a) 16 b) 16 c) 18

   d) 20 e) None of these

2. If x be real, find the max. and min. values of $\frac{x^2 - x + 1}{x^2 + x + 1}$

   a) 2, $\frac{1}{2}$ b) 3, $\frac{1}{3}$

   c) 4, $\frac{1}{4}$ d) 4, $\frac{1}{5}$

   d) 5, $\frac{1}{5}$ e) None of these

3. Find the max. value of $\frac{1}{1+x+x^2}$ for real values of x.

   a) $\frac{1}{3}$ b) $\frac{2}{3}$

   c) $\frac{4}{3}$ d) $\frac{5}{3}$ e) None

4. Find the min. value of $x + \frac{1}{x}$.

   a) 2 b) 3 c) 4 d) 5

   e) None

5. Find the greatest value of $x^2\ y^2$ when $x + 3y = 6c$.

   a) $\frac{49}{16}c^4$ b) $\frac{25}{16}c^4$

   c) $\frac{36}{16}c^4$ d) $\frac{81}{16}c^4$ e) None

Answers:

1. (b) 2. (b) 3. (c) 4. (a)

5. (d)