Quantitative Aptitude

PROBABILITY

Some Deflnitions:

Random Experiment: If an experiment, when repeated under indentical conditions, do not produce the same outcome every time but the outcome In a trial is one of the several possible outcomes, then it is known as a random experiment. In otherwords, an experiment whose outcomes cannot be predicted or determined in advance is called a random experiment.

For example, when we toss a coin, we are not sure if a head or tail will be obtained. So it is a random experiment.

Sample Space : The set S of all possible outcomes of an experiment or Observation is called a sample space provided no two or more of these outcomes can occur simultaneously and exactly one of the outcomes must occur whehever the experiment is performed.

For example, consider the 'experiment' of tossing two coins. If we arc interested in whether each coin falls heads (H) or tails (T), then the possible outcomes are

(H, H), (H, T), (T, H), (T, T)..........(i)

Event : An event is a subset of sample space. For example, for the sample space given by (i), the subset

A = {(H, H), (H, T), (T, H)} is the event that at least one head occurs.

Mutually Exclusive Events : If two or more events have no poinl in common i.e. if they cannot occur simultaneously, the events are said to be mutually exclusive.

Let two cards be drawn from a well-shuffled pact of 52 cards. Consider the following events ;

A = Gelting both red cards

B = Getting both black cards

Clearly, A and B are mutually exclusive events because two cards drawn cannot be both red and black at the same time.

Equally Likely Events : Two events are said to be equally likely if one of them cannot be expected to happen in preference to the other.

Exhaustive Events : A set of events is said to be exhaustive if no event outside this set occurs and at least one of these events must happen as a result of an experiment.

Thus when we toss a coin, either it must fall head or tail (the possibility of Standing on the edge is ruled out).

Favourable events : Let S be the sample space associnted with a random experiment and A be an event associaicd with the experiment. Then, elementary events belonging to A are known as favourable events tö the event A.

Consider the random experiment of throwing a pair of dice. Let A be the event "Getting 8 as the sum" Then,

A = ((2, 6), (6. 2), (4, 4), (5, 3), (3. 5)(.

Classical Definition of Probability : If there are n exhaustive, mutually exclusive and equally likely outcomes of an experiment and m of them are favourable to an event A, then the mathernatical probability of A is defined as Ihe ratio $\frac{m}{n}$.

Thus, $P(A) = \frac{\text{Favourable number of events}}{\text{Exhaustive number of events}} = \frac{m}{n}$

Clearly, $0 \le m \le n$. Therefore,

$0 \le \frac{m}{n} \le 1$

$\Rightarrow 0 \le P(A) \le 1$

The number of elementary events which will ensure the non-occurrence of A i.e. which ensure the occurrence of $\bar{A}$ is $(n - m)$. Therefore,

$P(\bar{A}) = \frac{n-m}{n} \Rightarrow P(\bar{A}) = 1 - \frac{m}{n}$

$\Rightarrow P(\bar{A}) = 1 - P(A) \Rightarrow P(A) + P(\bar{A}) = 1$

The odds in favour of occurrence of the event A are defined by $m : (n - m)$ i.e. $P(A) : P(\bar{A})$ and the odds against the occurrence of A are defined by $n - m : m$ i.e. $P(\bar{A}) : P(A)$.

Solved Examples

1. What is the probability that a leap year, selected at random, will contain 53 Sundays ?

(1) $\frac{1}{7}$ (2) $\frac{1}{3}$ (3) $\frac{2}{7}$ (4) $\frac{4}{7}$

Sol. In a leap year there are 366 days.

We have, 366 days = 52 weeks and 2 days.

The remaining 2 days can be

(i) Sunday and Monday (ii) Monday and Tuesday

(iii) Tuesday and Wednesday

(iv) Wednesday and Thursday

(v) Thursday and Friday (vi) Friday and Saturday

(vii) Saturday and Sunday

$\therefore$ Exhaustive number of cases = 7

Favourable number of cases = 2

$\therefore$ Required probability = $\frac{2}{7}$

2. Three dice are thrown together. Find the probability of getting a total of at least 6.

(1) $\frac{103}{216}$ (2) $\frac{103}{108}$ (3) $\frac{103}{36}$ (4) $\frac{36}{103}$

Sol. Since one die can be thrown in six ways to obtain any one of the six numbers marked on its six faces.

∴ Total number of elementary events

$= 6 \times 6 \times 6 = 216$

Let A be the event of getting a total of at least 6. Then $\bar{A}$ denotes the event of getting a total of less than 6 i.e. 3, 4, 5.

$\therefore \bar{A} = \{(1,1,1), (1,1,2), (1,2,1), (2,1,1), (1,1,3)$
$(1,3,1), (3,1,1), (1,2,2), (2,1,2), (2,2,1)\}$

So, favourable number of cases = 10

$\therefore P(\bar{A}) = \frac{10}{216} \Rightarrow 1 - P(A) = \frac{10}{216}$

$\Rightarrow P(A) = 1 - \frac{10}{216} = \frac{103}{108}$

3. One card is drawn from a pack of 52 cards, each of the 52 cards being equally likely to be drawn. Find the probability that the ball drawn is :

A. an ace

(1) $\frac{1}{13}$ (2) $\frac{1}{12}$ (3) $\frac{1}{14}$ (4) $\frac{1}{15}$

B. red

(1) $\frac{1}{3}$ (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) $\frac{1}{6}$

C. either red or king

(1) $\frac{2}{13}$ (2) $\frac{3}{13}$ (3) $\frac{5}{13}$ (4) $\frac{7}{13}$

D. red and a king

(1) $\frac{3}{26}$ (2) $\frac{5}{26}$ (3) $\frac{1}{26}$ (4) $\frac{7}{26}$

Sol. Out of 52 cards, one card can be drawn in ${}^{52}C_1$ ways.

∴ Total number of elementary events

$= {}^{52}C_1 = 52$

A. (1) There are four aces in a pack of 52 cards, out of which one ace can be drawn in ${}^{4}C_1$ ways.

∴ Favourable number of elementary events

$= {}^{4}C_1 = 4$

So, required probability $= \frac{4}{52} = \frac{1}{13}$

B. (2) There are 26 red cards, out of which one red card can be drawn in ${}^{26}C_1$ ways.

∴ Favourable number of elementary events

$= {}^{26}C_1 = 26.$

So, required probability $= \frac{26}{52} = \frac{1}{2}$

C. (4) There are 26 red cards including 2 red kings and there are 2 more kings. Therefore, there are 28 cards which are either red or king, out of 28 cards, one can be drawn in ${}^{28}C_1$ ways.

∴ Favourable number of elementary events

$= {}^{28}C_1 = 28.$

So, required probability $= \frac{28}{52} = \frac{7}{13}$.

D. (3) There are 2 cards which are red and king i.e., red kings.

∴ Favourable number of elementary events

$= {}^{2}C_1 = 2.$

So, required probability $= \frac{2}{52} = \frac{1}{26}$.

4. Four persons are to be chosen at random from a group of 3 men, 2 women and 4 children. Find the probability of selecting

A. 1 man, 1 woman and 2 children

(1) $\frac{2}{7}$ (2) $\frac{3}{7}$ (3) $\frac{4}{7}$ (4) $\frac{3}{7}$

B. exactly 2 children

(1) $\frac{9}{29}$ (2) $\frac{10}{21}$ (3) $\frac{12}{21}$ (4) $\frac{14}{19}$

C. 2 women

(1) $\frac{1}{5}$ (2) $\frac{1}{7}$ (3) $\frac{1}{6}$ (4) $\frac{1}{9}$

Sol. : There are 9 persons viz. 3 men, 2 women and 4 children. Out of these 9 persons 4 persons can be selected in ${}^{9}C_4 = 126$ ways.

∴ Total number of elementary events = 126.

A. (1) 1 man, 1 woman and 2 children can be selected in ${}^{3}C_1 \times {}^{2}C_1 \times {}^{4}C_1 = 36$ ways.

∴ Favourable number of elementary events = 36.

So, required probability $= \frac{36}{126} = \frac{2}{7}$.

B. (2) Exactly 2 children means 2 children and 2 from 3 men and 2 women. This can be done in ${}^{4}C_2 \times {}^{5}C_2$ ways.

∴ Favourable number of elementary events

$= {}^{4}C_2 \times {}^{5}C_2 = 60$

So, required probability $= \frac{60}{126} = \frac{10}{21}$.

C. (3) We have to select 4 persons of which 2 are women and the remaining 2 are chosen from 3 men and 4 children. This can be done in ${}^{2}C_2 \times {}^{7}C_2$ ways.

∴ Favourable number of elementary events

$= {}^{2}C_2 \times {}^{7}C_2 = 21$

So, required probability $= \frac{21}{126} = \frac{1}{6}$.

5. A word consists of 9 letters: 5 consonants and 4 vowels. Three letters are chosen at random. What is the Probability that more than one vowel will be selected ?

(1) $\frac{13}{42}$ (2) $\frac{17}{42}$ (3) $\frac{5}{42}$ (4) $\frac{3}{14}$

Sol. (2) Three letters can be chosen out of 9 letters in $^9C_3$ ways.

$\therefore$ Total number of elementary events = $^9C_3$

More than one vowels can be chosen in one of the following ways:

(i) 2 vowels and one consonant or (ii) 3 vowels.

So, favourable number of elementary events

$= {}^4C_2 \times {}^5C_1 + {}^4C_3$.

Hence, required probability $= \dfrac{^4C_2 \times {}^5C_1 + {}^4C_3}{^9C_3} = \dfrac{17}{42}$.

6. A bag contains 50 tickets numbered 1,2,3, ..., 50 of which five are drawn at random and arranged in ascending order of magnitude ($x_1 < x_2 < x_3 < x_4 < x_5$). Find the probability that $x_3 = 30$.

(1) $\dfrac{551}{15134}$ (2) $\dfrac{1}{2}$ (3) $\dfrac{552}{15379}$ (4) $\dfrac{1}{9}$

Sol. (1)

Five tickets out of 50 can be drawn in $^{50}C_5$ ways.

$\therefore$ Total number of elementary events = $^{50}C_5$

Since $x_1 < x_2 < x_3 < x_4 < x_5$ and $x_3 = 30$. Therefore, $x_1, x_2 < 30$ i.e., x_1 and x_2 should come from tickets numbered 1 to 29 and this may happen in $^{29}C_2$ ways. Remaining two i.e., $x_4, x_5 > 30$, should come from 20 tickets numbered from 31 to 50 in $^{20}C_2$ ways.

$\therefore$ Favourable number of elementary events

$= {}^{29}C_2 \times {}^{20}C_2$

Hence, required probability $= \dfrac{^{29}C_2 \times {}^{20}C_2}{^{50}C_5} = \dfrac{551}{15134}$.

7. A box contains 10 bulbs, of which just three are defective. If a random sample of five bulbs is drawn, find the probabilities that the sample contains

A. exactly one defective bulb,

(1) $\dfrac{5}{12}$ (2) $\dfrac{7}{12}$ (3) $\dfrac{3}{14}$ (4) $\dfrac{1}{12}$

B. exactly two defective bulbs,

(1) $\dfrac{1}{12}$ (2) $\dfrac{5}{12}$ (3) $\dfrac{7}{12}$ (4) $\dfrac{3}{14}$

C. no defective bulbs.

(1) $\dfrac{5}{12}$ (2) $\dfrac{7}{12}$ (3) $\dfrac{3}{14}$ (4) $\dfrac{1}{12}$

Sol.

Out of 10 bulbs 5 can be chosen in $^{10}C_5$ ways.

So, total number of elementary events = $^{10}C_5$

A. (1) There are 3 defective and 7 non-defective bulbs. The number of ways of selecting one defective bulb out of 3 and 4 non-defective out of 7 is $^3C_1 \times {}^7C_4$.

$\therefore$ Favourable number of elementary events

$= {}^3C_1 \times {}^7C_4$

So, required probability $= \dfrac{^3C_1 \times {}^7C_4}{^{10}C_5} = \dfrac{5}{12}$

B. (2) The number of ways of selecting 2 defective bulbs out of 3 and 3 non-defective bulbs out of 7 is $^3C_2 \times {}^7C_3$.

$\therefore$ Favourable number of elementary events

$= {}^3C_2 \times {}^7C_3$

So, required probability $= \dfrac{^3C_2 \times {}^7C_3}{^{10}C_5} = \dfrac{5}{12}$

C. (4) No defective bulbs means all non-defective bulbs. The number of ways of selecting all 5 non-defective bulbs out of 7 is $^7C_5$.

$\therefore$ Favourable number of elementary events = $^7C_5$

So, required probability $= \dfrac{^7C_5}{^{10}C_5} = \dfrac{1}{12}$.

8. An urn contains 9 red, 7 white and 4 black balls. If two balls are drawn at random, find the probability that:

(i) both the balls are red,

(ii) one ball is white

Sol. There are 20 balls in the bag out of which 2 balls can be drawn in $^{20}C_2$ ways.

So, total number of elementary events

$= {}^{20}C_2 = 190$

(i) There are 9 red balls out of which 2 balls can be drawn in $^9C_2$ ways.

$\therefore$ Favourable number of elementary events

$= {}^9C_2 = 36$.

So, required probability $= \dfrac{36}{190} = \dfrac{18}{95}$.

(ii) There are 7 white balls out of which one white can be drawn in $^7C_1$ ways. One ball from the remaining 13 balls can be drawn in $^{13}C_1$ ways. Therefore, one white and one other colour ball can be drawn in $^7C_1 \times {}^{13}C_1$ ways.

So, favourable number of elementary events

$= {}^7C_1 \times {}^{13}C_1$.

Hence, required probability $= \dfrac{^7C_1 \times {}^{13}C_1}{^{20}C_2} = \dfrac{91}{190}$

EVALUATE YOURSELF

1. Let $x = 33^n$. The index n is given a positive integral value at random. The probability that the value of x will have 3 in the units place is

(1) $\frac{1}{4}$ (2) $\frac{1}{2}$ (3) $\frac{1}{3}$ (4) None of these

2. Three dice are thrown simultaneously. The probability of getting a sum of 15 is

(1) $\frac{1}{72}$ (2) $\frac{5}{36}$ (3) $\frac{5}{72}$ (4) $\frac{5}{108}$

3. Three dice are thrown. The probability of getting a sum which is a perfect square is

(1) $\frac{2}{5}$ (2) $\frac{9}{20}$ (3) $\frac{1}{4}$ (4) $\frac{17}{108}$

4. The probability of getting a sum of 12 in four throws of an ordinary dice is

(1) $\frac{1}{6}\left(\frac{5}{6}\right)^3$ (2) $\left(\frac{5}{6}\right)^4$

(3) $\frac{1}{36}\left(\frac{5}{6}\right)^2$ (4) None of these

5. Three different numbers are selected at random from the set $A = \{1,2,3, \ldots, 10\}$. The probability that the product of two of the numbers is equal to the third is

(1) $\frac{3}{4}$ (2) $\frac{1}{40}$

(3) $\frac{1}{8}$ (4) None of these

6. A five-digit number is written down at random. The probability that the number is divisible by 5 and no two consecutive digits are identical, is

(1) $\frac{1}{5}$ (2) $\frac{1}{5}\cdot\left(\frac{9}{10}\right)^3$

(3) $\left(\frac{3}{5}\right)^4$ (4) None of these

7. If the letters of the word ATTEMPT are written down at random, the chance that all Ts are consecutive is

(1) $\frac{1}{42}$ (2) $\frac{6}{7}$ (3) $\frac{1}{7}$ (4) None of these

8. In a single cast with two dice the odds against drawing 7 is

(1) $\frac{1}{6}$ (2) $\frac{1}{12}$ (3) 5 : 1 (4) 1 : 5

9. 7 White balls and 3 black balls are placed in a row at random. The probability that no two black balls are adjacent is

(1) $\frac{1}{2}$ (2) $\frac{7}{15}$ (3) $\frac{2}{15}$ (4) $\frac{1}{3}$

10. 10 apples are distributed at random among 6 persons. The probability that at least one of them will receive none is

(1) $\frac{6}{143}$ (2) $\frac{^{14}C_4}{^{15}C_5}$ (3) $\frac{137}{143}$ (4) None of these

11. 4 gentlemen and 4 ladies take seats at random round a table. The probability that they are sitting alternately is

(1) $\frac{4}{35}$ (2) $\frac{1}{70}$ (3) $\frac{2}{35}$ (4) $\frac{1}{35}$

12. Five boys and three girls are seated at random in a row. The probability that no boy sits between two girls is

(1) $\frac{1}{56}$ (2) $\frac{1}{8}$ (3) $\frac{3}{28}$ (4) None of these

13. In a convex hexagon two diagonals are drawn at random. The probability that the diagonals intersect at an interior point of the hexagon is

(1) $\frac{5}{12}$ (2) $\frac{7}{12}$ (3) $\frac{2}{5}$ (4) None of these

14. 4 five-rupee coins, 3 two-rupee coins and 2 one-rupee coins are stacked together in a column at random. The probability that the coins of the same denomination are consecutive is

(1) $\frac{13}{9!}$ (2) $\frac{1}{210}$ (3) $\frac{1}{35}$ (4) None of these

15. Two cards are drawn at random from a pack of 52 cards. The probability of getting at least a spade and an ace is

(1) $\frac{1}{34}$ (2) $\frac{8}{221}$ (3) $\frac{1}{26}$ (4) $\frac{2}{51}$

16. One hundred identical coins each with probability p of showing up heads are tossed. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads on 51 coins; then the value of p is

(1) $\frac{1}{2}$ (2) $\frac{49}{101}$ (3) $\frac{50}{101}$ (4) $\frac{51}{101}$

17. Two dice are tossed. The probability that the total score is a prime number is

(1) $\frac{1}{6}$ (2) $\frac{5}{12}$ (3) $\frac{1}{2}$ (4) $\frac{7}{9}$

18. If the probability that A will live 15 years is $\frac{7}{8}$ and that B will live 15 years is $\frac{9}{10}$, then what is the probability that both will live after 15 years ?

(1) $\frac{1}{20}$ (2) $\frac{63}{80}$ (3) $\frac{1}{5}$ (4) None of these

19. When three coins are tossed together, the probability that all coins have the same face up is

(1) $\frac{1}{3}$ (2) $\frac{1}{6}$ (3) $\frac{1}{8}$ (4) $\frac{1}{4}$

20. If the probability of rain on any given day in Pune city is 50%, then what is the probability that it rains on exactly 3 days in a 5-day period ?

(1) 8/125 (2) 5/16 (3) 8/25 (4) 2/25

21. What is the probability of getting a sum of 9 from two throws of a dice ?

(1) 1/6 (2) 1/9 (3) 1/8 (4) 1/12