QUANTITATIVE APTITUDE

SIMPLIFICATION

Important Points:

1. Long Division Sum : In a division sum we have four quantities : Dividend, Divisor, Quotient and Remainder. These quantities bear the following relation :

Dividend = Divisor x Quotient + Remainder

In symbolic representation,

Divisor) Dividend(Quotient

Remainder

(a) Dividend : The number which is to be divided is called dividend.

(b) Divisor : The number by which the dividend is divided is called the divisor.

(c) Quotient: The number of times the divisor is contained in the dividend is called the quotient.

(d) Remainder : The number left over after division is called the remainder.

2. Number at the Unit Place of N" where N and n are positive integers.

(a) In N^n, if *N* contains anyone of 0. 1, 5 or 6 at the unit place, for any value of *n,* N^n has the same digit at the unit place.

(b) In N^n, if JV contains anyone of 2, 3, 7 or 8, we divide *n* by 4 and substitute the number *n* by the remainder that may be either 0 or 1 or 2 or 3. The digit at the unit place of JV' will be same as that of N^n where r is remainder.

(c) In N^n, if AT contains 4 at the unit place, then N^n will have 4 or 6 at the unit place according as nis odd or even.

(d) In N^n, if JV contains 9 at the unit place, then N^n will have 9 or 1 at the unit place according as nis odd or even.

3. Various Types of Numbers :

(a) Natural Numbers and Whole Numbers. Counting numbers are known as Natural numbers. Thus, N = {1, 2, 3, 4, 5, ...} denotes the set of all natural numbers. The set, W = {0, 1, 2, 3, 4, 5 ....} is called the set of Whole numbers.

(b) Integers: All counting numbers, together with their negatives and 0 constitute the set

I = {.... - 3, - 2, - 1, 0, 1, 2, ...} of integers.

As an example, 1857 is a positive integer, - 1857 is a negative integer and 0 is an integer, which is neither positive nor negative.

(c) **Fractions :** The numbers of the form, $\frac{p}{q}$, where *p* and *q* are natural numbers, are called fractions, e.g. $\frac{3}{5}, \frac{12}{7}$ etc. In particular, each natural number is a fraction, since we may express 5 as (5/1).

A fraction in which the numerator and denominator have no common factors, is said to be in the simplest form.

A fraction in which the numerator is less than the denominator, is known as a proper fraction, otherwise it is improper.

(d) **Rational Numbers** : The numbers of the form (p/q), where p and q are integers and $q \neq 0$, are called rational numbers.

(e) **Irrational Numbers** : A number, which when expressed in decimal form, is expressible neither in terminating decimal nor in repeating decimal, is known as an irrational number, e.g., $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi, e$ etc.

(f) **Real Numbers** : Totality of rationals and irrationals forms the set R of all real numbers.

4. Recurring Decimals : If in a decimal, some figures or a set of figures repreat continually, we call it a recurring decimal. The set of repeated figures is called period. For example,

$$\frac{22}{7} = 3.142857142857 \ldots\ldots$$

$$= 3.\overline{142857} \text{ or } 3.\dot{1}42857\dot{}$$

$$\frac{2}{3} = 0.6666 \ldots\ldots = 0.\overline{6} \text{ or } 0.\dot{6}$$

Rules for Converting Recurring Decimals into Vulgar Fractions: Write the repeated figures only once in the numerator and take as many nines in the denominator as is the number of repeating figures. For example,

$$0.666 \ldots\ldots = 0.\dot{6} = \frac{6}{9} = \frac{2}{3}$$

$$3.\dot{5} = 3 + 0.\dot{5} = 3 + \frac{5}{9} = 3\frac{5}{9}$$

Rule for Converting A Mixed Recurring Decimal into a Vulgar Fraction: Recurring decimals like 0.59 , 0.387 are mixed recurring decimals as **5** and **3** donot bear bar or dpt. Form a fraction in which the numerator is the difference between the number formed by all the digits after the decimal point, taking the repeated digits only once and that formed by the digits, which are not repeated and the denominator is the number formed by as many nines as there are repeating digits followed by as many zeros as is the number of non-repeating digits.

For example,

$$0.1777 \ldots\ldots = 0.1\dot{7} = \frac{17.1}{90} = \frac{16}{90} = \frac{8}{45}$$

$$0.12\overline{57} = \frac{1257-12}{9900} = \frac{1245}{9900}$$

5. VBODMAS Rule : In simplifying an expression containing fractions, we strictly maintain the order of operations in the following manner :

(a) Vinculum or Bar : First of all, vinculum or bar must be removed. For example,

$-\overline{5-7} = -(-2) = 2$ where as $-5-7 = -12$

(b) After removing the bar, the brackets must be operated on strictly in the order ( ), { } and [ ].

(c) After removing brackets, we must use the following operations strictly in the order given below.

(i) of (ii) Division (iii) Multiplication (iv) Addition (v) Subtraction.

Remember the rule VBODMAS where V, B, O, D, M, A, S stand for Vinculum, Bracket, Of , Division, Multiplication, Addition and Subtraction respectively.

6. Algebraic Identities :

(a) $(a+b)^2 = a^2 + 2ab + b^2$

(b) $(a-b)^2 = a^2 - 2ab + b^2$

(c) $(a^2-b^2) = (a+b)(a-b)$

(d) $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$
$= a^3 + b^3 + 3ab(a+b)$

(e) $(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$
$= a^3 - b^3 - 3ab(a-b)$

(f) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

(g) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

(h) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

(i) $(a+b)^2 = (a-b)^2 + 4ab$

(j) $(a-b)^2 = (a+b)^2 - 4ab$

(k) $a^3 + b^3 + c^3 - 3abc$
$= (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ac)$
If $a+b+c = 0$, then $a^3 + b^3 + c^3 = 3abc$

SOLVED EXAMPLES

1. Two different numbers when divided by a certain divisor leave remainders 547 and 349 respectively. When the sum of the two numbers is divided by the same divisor, the remainder is 211. Find the divisor.

[SSC Graduate Level (UDC) Main Exam, 21.02.2009]

Sol. Let the common divisor be D and Q_1 and Q_2 be the respective quotients.

$\therefore$ First number $(x) = Q_1 \times D + 547$

Second number $(y) = Q_2 \times D + 349$

$\therefore$ $x + y = (Q_1 + Q_2)D + 547 + 349$

$= (Q_1 + Q_2)D + 896$

$= (Q_1 + Q_2)D + 685 + 211$

$\because$ $547 < D$ and $349 < D$

Since each of the remainders is less than the divisor, the latter can be subtracted only once from the sum of the former two.

$\therefore$ Divisor = $547 + 349 - 211 = 685$

2. Simplify : $\frac{3}{4} + \frac{5}{36} + \frac{7}{144} + \ldots\ldots\ldots + \frac{17}{5184} + \frac{19}{8100}$

[SSC Graduate Level (UDC) Main Exam, 21.02.2009]

Sol. Here, $\frac{3}{4} = \frac{4-1}{1\times 4} = 1 - \frac{1}{4}$

$\frac{5}{36} = \frac{9-4}{4\times 9} = \frac{1}{4} - \frac{1}{9}$

$\frac{7}{144} = \frac{16-9}{16\times 9} = \frac{1}{9} - \frac{1}{16}$

.....

.....

$\frac{19}{8100} = \frac{100-81}{81\times 100} = \frac{1}{81} - \frac{1}{100}$

$\therefore$ Expression

$= 1 - \frac{1}{4} + \frac{1}{4} - \frac{1}{9} + \frac{1}{9} - \frac{1}{16} + \ldots\ldots + \frac{1}{81} - \frac{1}{100}$

$= 1 - \frac{1}{100} = \frac{100-1}{100} = \frac{99}{100}$

3. On dividing a number by 4, the sum of quotient and remainder is 8 and sum of their squares is 34. Find the number.

Sol. : Let the quotient be Q and the remainder R.

$\therefore$ Number $= 4 \times Q + R$

According to the question

$Q + R = 8$

and $Q^2 + R^2 = 34$

Now, $(Q+R)^2 = Q^2 + R^2 + 2QR$

$\Rightarrow$ $(8)^2 = 34 + 2QR$

$\Rightarrow$ $64 = 34 + 2QR$

$\Rightarrow$ $2QR = 64 - 34 = 30$ ....(i)

Now, $(Q-R)^2 = Q^2 + R^2 - 2QR$

$= 34 - 30$ [From (i)

$= 4$

$\therefore$ $Q - R = \sqrt{4} = 2$

Thus, we have

$Q + R = 8$

$Q - R = 2$

On adding both of these, we have

$2Q = 10 \Rightarrow Q = \frac{10}{2} = 5$

$\therefore$ $R = 8 - Q = 8 - 5 = 3$

$\therefore$ Number $= 4Q + R = 4 \times 5 + 3 = 20 + 3 = 23$

4. A shopkeeper buys a number of books for Rs. 80. If he had bought 4 more books for the same amount, each book would have cost Re. 1 less. How many books did he buy?

Sol. : Let the number of books bought by the shopkeeper be x.

Then, cost of x books = Rs. 80

$\therefore$ Cost of one book = Rs. $\left(\frac{80}{x}\right)$

If the number of books bought is $x + 4$.

Then, cost of one book = Rs. $\left(\frac{80}{x+4}\right)$

As per the given information, the cost of one book is reduced by one rupee.

$\therefore \frac{80}{x} - \frac{80}{x+4} = 1$ or, $80\left(\frac{1}{x} - \frac{1}{x+4}\right) = 1$

or, $80\left(\frac{x+4-x}{x(x+4)}\right) = 1$ or, $\frac{320}{x^2+4x} = 1$

or, $x^2 + 4x - 320 = 0$

or, $x^2 + 20x - 16x - 320 = 0$

or, $x(x+20) - 16(x+20) = 0$

or, $(x+20)(x-16) = 0$

or, $x = -20$ or 16

But x cannot be negative.

Hence, the number of books = 16

5. If $\sqrt{4096} = 64$, then the value of

$\sqrt{40.96} + \sqrt{0.4096} + \sqrt{0.004096} + \sqrt{0.00004096}$

upto two places of decimals is

(1) 7.09 (2) 7.10 (3) 7.1104 (4) 7.12

Sol. : Given expression

$$= \sqrt{\frac{4096}{100}} + \sqrt{\frac{4096}{10000}} + \sqrt{\frac{4096}{1000000}} + \sqrt{\frac{4096}{100000000}}$$

$$= \frac{64}{10} + \frac{64}{100} + \frac{64}{1000} + \frac{64}{10000}$$

$= 6.4 + 0.64 + 0.064 + 0.0064 = 7.1104$

$\therefore$ Option (3) is correct.

6. The value of $\left\{\frac{(2.3)^3 - 0.027}{(2.3)^2 + 0.69 + 0.09}\right\}$ is

(1) 2.6 (2) 2 (3) 2.33 (4) 2.27

Sol. : Let $2.3 = a$ and $0.3 = b$.

$\therefore$ Expression

$$= \frac{a^3 - b^3}{a^2 + ab + b^2} = \frac{(a-b)(a^2 + ab + b^2)}{a^2 + ab + b^2}$$

$= a - b = 2.3 - 0.3 = 2$

$\therefore$ Option (2) is correct.

Directions (7–8) : What should come in place of th question mark (?) in the following questions ?

[UCO Bank PO Exam, 22.03.200

7. 15.5% of 646 + 24.5% of 298 = ?

(1) 184.22 (2) 173.14 (3) 168.26

(4) 137.41 (5) None of these

Sol. (2) $? = \frac{646 \times 15.5}{100} + \frac{298 \times 24.5}{100}$

$= 100.13 + 73.01 = 173.14$

8. $12.25 \times ? \times 21.6 = 3545.64$

(1) 14.8 (2) 12.6 (3) 15.8

(4) 13.4 (5) None of these

Sol. (4) $? = \frac{3545.64}{12.25 \times 21.6} = 13.4$

Directions (9–10) : What should come in place of the question mark (?) in the following questions ?

[NABARD Bank Officer Exam, 29.03.2009]

9. $\sqrt[3]{?} = (756 \times 67) \div 804$

(1) 195112 (2) 250047 (3) 226981

(4) 274625 (5) None of these

Sol. (2) $\sqrt[3]{?} = \frac{756 \times 67}{804} = 63$

$\therefore ? = (63)^3 = 63 \times 63 \times 63$

$= 250047$

10. 0.3+3+3.33+3.3+3.03+333 = ?

(1) 375.66 (2) 345.99 (3) 375.93

(4) 355.96 (5) None of these

Sol. (5) $? = 0.3 + 3 + 3.33 + 3.3 + 3.03 + 333 = 345.96$

11. Simplify :

$$\frac{1664}{1408}\text{of}\left\{\frac{7}{10}\text{of Rs.}3\frac{3}{14}+6\frac{2}{3}\text{of Rs.}3\frac{3}{80}-4\frac{13}{31}\text{of Rs. 3.10P}\right\}$$

12. Simplify :

$$\left\{\frac{1\frac{2}{3}\div1\frac{3}{4}}{1\frac{4}{5}\div1\frac{5}{6}}\div\frac{1\frac{6}{7}\div1\frac{7}{8}}{1\frac{8}{9}\div1\frac{9}{10}}\right\}\div\left\{\frac{\frac{1}{7}\div\frac{1}{9}}{\frac{1}{11}\div\frac{1}{13}}\div\frac{\frac{1}{17}\div\frac{1}{18}}{\frac{1}{19}\div\frac{1}{20}}\right\}$$

13. Simplify :

$$\frac{1}{7\frac{1}{4}+6\frac{6}{11}}\div\left(\frac{3}{13}-\frac{2}{9}\right)-\left(\frac{13}{3}+\frac{1}{6}\right)\div\frac{2}{3}\text{ of }\frac{3}{8}\text{of }63$$

14. Find the value of

$$\left\{\frac{3\frac{1}{2}\text{of }5\frac{5}{8}}{2\frac{5}{6}\text{of }3\frac{1}{3}}\div\frac{2\frac{2}{11}\text{of }\frac{11}{12}}{3\frac{1}{9}\text{of }7\frac{5}{9}}\right\}\text{of }\frac{\text{Rs. }1.70P}{\text{Rs}5.50P}\text{of }\frac{2m.43cm.}{5m.85cm}$$

of 24 weeks 4 days 19 hours.

15 Compute : $1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\frac{1}{5}}}}$

Objective Type Questions

16. If $(67^{67}+67)$ is divided by 68, the remainder is

(1) 1 (2) 67
(3) 63 (4) 66

17. If * is a digit such that 5824* is divisible by 11, then * equals

(1) 2 (2) 3
(3) 5 (4) 6

18. The square of a natural number subtracted from its cube is 48. The number is

(1) 8 (2) 6
(3) 5 (4) 4

19. How many digits in all are required to write numbers from 1 to 50 ?

(1) 100 (2) 92
(3) 91 (4) 50

20. If 25^{25} is divided by 26, the remainder is

(1) 1 (2) 2
(3) 24 (4) 25

21. The sum $(5^3 + 6^3 + ... + 10^3)$ is equal to

(1) 2295 (2) 2425
(3) 2495 (4) 2925

22. The sum 9 + 16 + 25 + 36 + ... + 100 is equal to

(1) 350 (2) 380
(3) 400 (4) 420

23; The ratio between two numbers is 3 : 4 and their um is 420. The greater of the two numbers is

(1) 175 (2) 200
(3)240 (4) 315

24. The digit in the unit's place of

$[(251)^{98} + (21.!\quad - (106)^{100} + (705)^{35} - 16^4 + 259]$

is

(1) 1 (2) 4
(3) 5 (4) 6

25. A number when divided by the sum of 555 and 445 gives two times their difference as quotient and 30 as the remainder. The number is

(1) 220030 (2) 22030
(3) 1220 (4) 1250

26. If a number is divisible by both 11 and 13, then it must be necessarily

(1) divisible by (11 + 13) (2) divisible by (13 - 11)
(3) divisible by (11 x 13) (4) 429

27. If doubling a number and adding 20 to the result gives the same answer as multiplying the number by 8 and taking away 4 from the product, the number is

(1) 2 (2) 3
(3)4 (4) 6

28. A number when divided by 6 leaves a remainder 3. When the square of the same number is divided by 6, then remainder is

(DO (2) 1
(3) 2 (4) 3

29. The sum of the first forty-five numbers is divisible by

(1) 21 (2) 23
(3) 44 (4) 46

30. How many numbers from 1000 are multiples of both 10 and 13 ?

(1) 9 (2) 8
(3) 6 (4) 7

ANSWERS

1. 20 days **2.** 75
3. 789 **4.** 39
5. 24, 30, 9, 81 **6.** 72
7. 375 **8.** 53, 58, 63
9. 7, 11 **10.** $2^{44}-1$
11. Rs. 10.40 **12.** $\frac{875}{972}$
13. $8\frac{830}{4249}$
14. 79 weeks 1 day 22.83 hrs.
15. $1\frac{68}{57}$

16.(4) **17.**(3) **18.**(4) **19.**(3) **20.**(4)
21.(4) **22.**(2) **23.**(3) **24.**(2) **25.**(1)
26.(3) **27.**(3) **28.**(4) **29.**(2) **30.**(4)