VOLUME & SURFACE AREA

We know that every real object occupies some Space. Itis usually specified by iis three dimensions—length, breadth and depth (or height or thickness). It may be a solid or a hollow body. In case of circular, cylindiical and spherical object the specifying dimensions may change to radius, angle etc. The amount of space occupied by the object is called its volume. Its unit of measurement is m^3, cm^3, $(inches)^3$ etc. The area of the surfaces (plane/Curved) of the object is called its *surface area.*

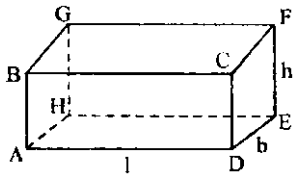
A 2-dimensional figure is a special case or a limiting case of a 3-dimensional object in vvhich its third dimension i.e., deplh (or thickness) is negligible in comparison to its other two dimensions i.e., length and breadth.

We illustrate below some important 3-dimensional object and also write formulae connected with them.

In general, we will use l = length, h = breadth, h = height. *w* = width, r= radius, *D* = diameter, P= perimefer, A = area, V = volume.

Cuboid

It is made up of 6 rectangular faces. All sides/edges/ faces meet at right angle pair of opposite faces are equal.



Volume : $V = lbh$ cubic units

Total Surface Area : $A = 2(lb + bh + lh)$ Square units

Facc Diagonals : $AC = BD = GE = FH = \sqrt{l^2 + h^2}$ units

$DF = CE = AG = BH = \sqrt{b^2 + h^2}$ units

$AE = DH = BF = CG = \sqrt{l^2 + b^2}$ units

Body Diagonal or Diagonal of the Cuboid :

$AF = BE = DG = CH = \sqrt{l^2 + b^2 + h^2}$ units.

Cube

It is a special case of cuboid in which $l = b = h = a$ units

Volume $= a^3$ cubic units

Total Surface Area $= 6a^2$ sq. units

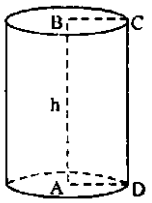
$$\text{Volume} = \left(\sqrt{\frac{\text{Surface Area}}{6}}\right)^3 \text{ cubic units}$$

Face Diagonal $= \sqrt{2}\ a$ units

Body Diagonal or Diagonal $= \sqrt{3}\ a$ units.

Cylinder

It is formed by rotating one side of a rectangle about its opposite side (keeping fixed).



r = radius of the cylinder

h= height of the cylinder or length of the cylinder.

Heve *ADCD* is the generating rectangle for the cylinder.

Volume $= \pi r^2 h$ cu. units

Area of the Curved Surface $= 2\pi rh$ sq. units

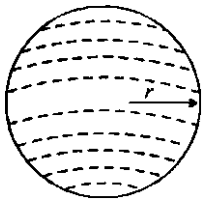
Area of the Base = Area of the top $= \pi r^2$ sq. units

Total Surface Area $= (2\pi rh + 2\pi r^2)$ sq. units

$= 2\pi r(h + r)$ sq. units

Sphere

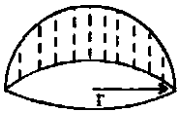
Radius of the Sphere= r units



/olume $= \frac{4}{3}\pi r^3$ cu. units

Surface Area= $4\pi r^2$ sq. units

Hemisphere



Volume $= \frac{2}{3}\pi r^3$ cu. units

Area of the Curved Surface $= 2\pi r^2$ sq. units

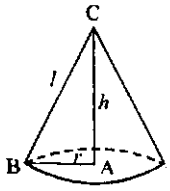
Total Surface Area= $3\pi r^2$ sq. units

Cone

A cone is formed by rotating a right-angled triangle around its height. (or right-circular cone)

l= slant height, h = height, r = radius

$l = \sqrt{r^2 + h^2}$



$V = \frac{1}{3}\pi r^2 h$ cu. units.

Area of the Curved Surface

$= \pi rl = \pi r\sqrt{r^2 + h^2}$ sq. units.

Area of the Base= πr^2 sq. units

Total Surface Area of the Cone $= (\pi rl + \pi r^2) = \pi r (l + r)$ sq. units

Frustum of a Right Circular Cone

If a cone is cut by a plane parallel to the base of the cone, the lower part is called the frustum of the cone.

s= Slant height of the frustum

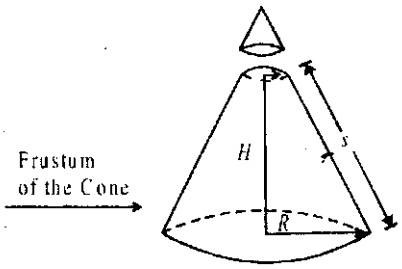
R = Radius of the bigger end (Bottom here)

r= Radius of the smaller end (Top here)

h= Height of the frustum

$s = \sqrt{h^2 + (R - r)^2}$

$V = \frac{\pi h}{3}(R^2 + r^2 + Rr)$ cu. units.



Area of the Curved or Slant Surface = $\pi (R + r)\ s$ sq. units

Total Surface Area of the Frustum = $\pi [(R^2 + r^2) + s$ (R + r)l sq. units.

Density

Density of an object is its mass per unit volume.

Mass = Volume x Density

Assumption : When an object is made to change shape or/and size either by melting or by some other means, we will assume that its density does not change. Therefore, if the problem involves a fixed mass object changing its shape/size. We can say that its volume remains same on the basis of above assumption.

Let's consider some important questions asked in previous exam.

1. A cylindrical tank of diameter 35 cm is füll of water. If 11 litres water is taken out from the tank, find the drop in the water level in the tank. (Use $\pi = \frac{22}{7}$)

[SSC Graduate Level (Asstt. Grade) Main Exam. 2004]

Sol. Let r = Radius of the cylindrical tank = $\frac{35}{2}$ cm

Let the initial and new heights of water level be h and li' respectively.

According to the question,

$\pi r^2 h - \pi r^2 h' = 11 \Rightarrow \pi r^2 (h - h') = 11 \times 1000$ c

$\Rightarrow h - h' = \frac{11000}{\pi r^2} = \frac{11000}{\frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}}$

$= \frac{11000 \times 7 \times 2 \times 2}{22 \times 35 \times 35} = \frac{80}{7} = 11.428$ cm.

∴ Drop in water level = 11.428cm

2. A tank is of the shape of a cuboid whose length is 7.2 m. and breadth is 2.5 m. Water flows into it througha pipe whose cross-section is 5 cm. x 3 cm. at the rate of 10 m per second. Find the height to which water level will rise in the tank in 40 minutes.

[SSC Graduate Level (UDC) Main Exam, 2004]

Sol. Volume of the water that flows in 1 second = (lOx

$0.05 \times 0.03)\text{m}^3 = \left(\frac{10 \times 5 \times 3}{100 \times 100}\right)\text{m}^3 = \left(\frac{15}{1000}\right)\text{m}^3$

Volume of the water that flows in 40 minutes

$= \left(\frac{15}{1000} \times 60 \times 40\right) = 36\ \text{m}^3$

Area of the base of cu,boid = 7.2 x 2.5 m^2

∴ Height of water level = $\frac{36}{7.2 \times 2.5}$ m = 2 m

3. The diametcrs of the ends of a dustbin 25 cm. high, which is in the shape of a frustum of a cone, are 30 cm. and 10 cm. Determine its capacity and surface area. (Take $\pi = \frac{22}{7}$)

[SSC Graduate Level (UDC) Main Exam, 2004]

Sol. For frustum of a cone,

let height (h) = 24 cm.

Upper radius (R) = 15 cm.

Lower radius (r)= 5 cm.

∴ Slant height $l = \sqrt{h^2 + (R - r)^2} = \sqrt{24^2 + (15 - 5)^2}$

$= \sqrt{576 + 100} = \sqrt{676} = 26$ cm.

∴ Capacity of the dustbin $= \frac{\pi h}{3}\left(R^2 + r^2 + Rr\right)$

$= \frac{22}{7 \times 3} \times 24\left(15^2 + 5^2 + 15 \times 5\right) = \frac{176}{7}(225 + 25 + 75)$

$= \frac{176}{7} \times 325 = \frac{57200}{7} = 8171.43\ \text{cm}^3.$

Surface area = $\pi[(R^2 + r^2) + l\,(R + r)]$

$= \frac{22}{7}[(15^2 + 5^2) + 26\,(15 + 5)]$

$= \frac{22}{7}$ 1(225 + 25) + 26 x 20]

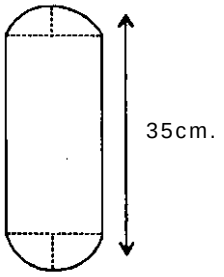
$= \frac{22}{7}$ x (250 + 520)

$= \frac{22}{7}$ x 770 = 2420 cm^2.

4. A solid is in the form of a right circular cylinder with hemispherical ends. The total length of the soild is 35 cm. The diameter of the cylinder is $\frac{1}{4}$ th of its height. Find the volume and total surface area of the soild. $\left(\text{Use } \pi = \frac{22}{7}\right)$

[SSC Graduate Level (Asstt. Grade) Main Exam. 2002]

Sol.



Let r cm. be the radius and h cm. be the height of the cylinder then,

$r = \frac{h}{8}(\text{Given})$ and $h = 35 - 2r = 35 - 2 \times \frac{h}{8}$

$\Rightarrow h + \frac{h}{4} = 35 \Rightarrow \frac{5h}{4} = 35$

$\Rightarrow h = \frac{35 \times 4}{5} = 28$ cm. $\therefore r = \frac{h}{8} = \frac{28}{8} = \frac{7}{2}$ cm.

Also, radius of hemisphere $= \frac{7}{2}$ cm.

Now, volume of the solid = (Volume of the cylinder) + (Volume of two hemispheres)

$= \left[\pi r^2 h + 2 \times \frac{2}{3}\pi r^3\right] cm^3 = \pi r^2 \left(h + \frac{4r}{3}\right) cm^3$

$= \pi r^2 \left(h + \frac{4r}{3}\right) cm^3 = \left[\frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times \left(28 + \frac{4}{3} \times \frac{7}{2}\right)\right] cm^3$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{98}{3} = 1257.67 cm^3$

Sufrace area of the solid = curved Surface area of cylinder + Surface area of two hemispheres.

$= (2\pi rh + 2 \times 2\pi r^2)\ cm^2$.

$= 2\pi r\ (h + 2r)\ cm^2$.

$= 2 \times \frac{22}{7} \times \frac{7}{2}\left(28 + 2 \times \frac{7}{2}\right) cm^2$

$= \left(2 \times \frac{22}{7} \times \frac{7}{2} \times 35\right) cm^2 = 770\ cm^2$

5. The ratio of the radii of the bases of two cylinders is 2 : 3 and the ratio of their heights is 5 : 3 respectively. The ratio of their volumes will be
(1) 27 : 20 (2) 20 : 27
(3) 9 : 4 (4) 4 : 9

[SSC Combined Graduate Level Prelim. Exam, 2005, (First sitting)]

Sol. $\frac{\text{Volume of first cylinder}}{\text{Volume of cylinder}} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2}$

$= \left(\frac{r_1}{r_2}\right)^2 \times \left(\frac{h_1}{h_2}\right) = \left(\frac{2}{3}\right)^2 \times \frac{5}{3} = \frac{4}{9} \times \frac{5}{3} = 20 : 27$

$\therefore$ Option (2) is the correct answer.

6. The volume of a cuboid is twice the volume of a cube. If the dimensions of the cuboid be 9 cm, 8 cm and 6 cm, then the total surface area of cube will be
(1) 72 cm^2 (2) 216 cm^2
(3) 432 cm^2 (4) 108 cm^2

[SSC Combined Graduate Level Prelim. Exam, 2005, (Second sitting)]

Sol Volume of Cuboid = 9 x 8 x 6 = 432 cm^3
According to the question.

Volume of cube = $\frac{432}{2}$ = 216 cm^3

$\therefore$ Edge of eube = $\sqrt{216}$ = 6 cm
$\therefore$ Total surface area of the eube
= 6 x $(\text{edge})^2$ = 6 x 6^2 = 216 cm^2
$\therefore$ Option (2) is the correct answer,

7. The volume of the metal of a cylindrical pipe is 748 cm^3. The length of the pipe is 14cm and its external radius is 9 cm. Its thickness is $\left(\text{Take } \pi = \frac{22}{7}\right)$

(1)1 cm (2)5.2 cm
(3)2.3 cm (4)3.7 cm

[SSC Combined Graduate Level Prelim. Exam. 27.07.2008, (First sitting)]

Sol. (1) Let the thickness of the pipe = x cm
$\therefore$ If the external radius = 9 cm
then, in radius = (9 - *x)* cm
According to the question,
$\pi \times 9^2 \times 14 - \pi \times 14 \times (9 - x)^2 = 748$
$\Rightarrow \pi \times 14\ (81 - (81 + x^2 - 18x)) = 748$

$\Rightarrow \pi \times 14 (-x^2 + 18x) = 748$

$\Rightarrow -x^2 + 18x = \frac{748}{\pi \times 14} = \frac{748 \times 7}{22 \times 14}$

$\Rightarrow -x^2 + 18x = 17$

$\Rightarrow x^2 - 18x + 17 = 0$

$\Rightarrow x^2 - 17x - x + 17 = 0$

$\Rightarrow x(x - 17) - 1(x - 17) = 0$

$(x - 1)(x - 17) = 0$

$\Rightarrow x = 1$ or 17 but

$x = 17$ is inadmissible

8. Water flows through a cylindrical pipe, whose radius is 7 cm, at 5 metres per second. The time, it takes to Pill an empty water tank, with height 1.54 metres and area of the base (3 x 5) Square metres, is $\left(\text{take } \pi = \frac{22}{7}\right)$

(1) 6 minutes (2) 5 minutes

(3) 10 minutes (4) 9 minutes

[SSC Combined Graduate Level Prelim. Exam, 27.07.2008, (Second sitting)]

Sol. (2) Volume of water flowing per second $= \pi r^2 h$

$= \frac{22}{7} \times \frac{7 \times 7 \times 5}{10000}$ metre^3 = 0.077 metre^3

Volume of the tank = 3 × 5 × 1.54 metre^3

= 23.1 metre^3

$\therefore$ Required time $= \frac{23.1}{0.077} = 300$ second

= 5 Minutes

9. Water is flowing at the rate of 5 km/hr through a cylindrical pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Determine the time in which the level of water in the tank will rise by 7 cm.

$\left(\text{Use } \pi = \frac{22}{7}\right)$

[SSC Section Officer (Audit) Exam, 30.11.2008]

Sol. Rate of flow of water = 5 kmph

$= \frac{5 \times 5}{18}$ m/sec. $= \frac{25}{18}$ m/sec.

Volume of water flowing per second through the cylindrical pipe = $\pi r^2 h$

$= \frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times \frac{25}{18} = \frac{77}{3600}$ cubic metre

Volume of water to be filled in the rectangular tank

$= 50 \times 44 \times \frac{7}{100} = 154$ cubic metre

$\therefore$ Required time $= \frac{154}{\frac{77}{3600}} = \frac{154 \times 3600}{77}$

= 7200 seconds = 2 hours

TEST YOURSELF

1. The volume of a cube is 1000 cu. cm. Find its total surface area.

2. Find the length of the longest pole that can be placed in a room 12 metres long, 9 metres broad and 8 metres high.

3. Two cubes, each of volume 512 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

4. A fieldis 150mlongand 100 m wide. A plot (outside the field) 50 m long and 30 m wide is dug to a depth of 8 m and earth taken out from the plot is spread evenly in the field. By how much is the level of field raised?

5. A rectangular tank is 80 m long and 25 m broad. Water flows into il through a pipe whose cross-section is 25 cm^2, at the rate of 16 km per hour. How much will the level of water rise in the tank in 45 minutes?

6 The dimensions of a rectangular box are in the ratio of 2 : 3 : 4 and the difference between the cost of covering it with sheet of paper at the rates of Rs. 8 and Rs. 9.50 per m^2 is Rs. 1248. Find the dimensions of the box.

7. A metallic sheet is of rectangular shape with dimensions 48 cm. x 36 cm. From each one of its corners. a Square of 8 cm is cut off. An open box is made of the remaining sheet. Find the volume of the box.

6. The circumference of the base of a right circular cylinder is 13.2 metres and the sum of its radius and the height is 3 metres. Find the radius and total surface area of the cylinder.

9. The height of a right circular cylinder is 6 metres and three times the sum of the areas of its two end faces is twice the area of its curved surface, find the radius of its base.

10 96 equal circular plates, each half a centimetre thick. are placed one above another to form a right circular cylinder of volume 7392 cu. cm. find the radius of each circular plate.

11. A garden roller made of iron is 32 cm long and its external circumference is 44 cm; the difference between its external and internal radii is 1 cm. Find the weight of the roller, given that a cubic centimetre of iron weighs 7.861 gms.

12. The external and the internal radii of a reservoir having concrete wall, 7 metres high are 5 metres and 4 metres respectively. Find the volume of concrete used in its construction and quantity of water it can hold.

13 A hollow cylinder whose internal diameter is 20 cm is closed at one end and is partly filled with waler. By how much does the level of water in the cylinder rise when a cube, each edge of which measures 11 cm is completely immersed in it?

14 A copper wire of diameter 6 mm is evenly wrapped on the cylinder of length 18 cm and diameter 49 cm to cover its whole surface. Find the length of the wire.

15 Find the number of coins 1.5 cm in diameter and 0.2 cm thick to be melted to form a right circular cylinder whose height is 10 cm and diameter 4.5 cm.

OBJECTIVE TYPE QUESTIONS

16. The breath of a cuboid is twice its height and half its length. Ifthe volume of the couboid is 512 m^2, then the length of the couboid is
(1) 12 m (2) 16 m
(3) 232 m (4) 20 m

17. The volume of a rectangular block of stone is 10368 dm^2. Its dimensions are in the ratio of 3 : 2 :1. If its entire surface is polished at 2 paise per dm^2, then the total cost will be
(1) Rs. 31.50 (2) Rs. 63.00
(3) Rs. 63.36 (4) Rs 31.68

18. One cubic metre piece of copper is melted and re-cast into a Square cross-section bar 36m long. An exact cube is cut off from this bar. If 1 cubic metre of copper costs Rs. 108, then the cost of this cube is
(1)50 paise (2) 75 paise
(3) One rupee (4) 1.50 rupees

19. Ifthe volumes of two cubes are in the ratio of 8:27, then the ratio of their edges is
(1) 1 : 3 (2) 2 : 3
(3) 4 : 3 (4) 5 : 3

20. The length of the longest rod that can be placed in a room 30m long, 24 m broad and 18 m high is
(1) 30 m (2) $15\sqrt{2}$ m
(3) $30\sqrt{2}$ m (4) 60 m

21. Ifthe volume of a cube is 216 cm^2, then the surface area of the cube will be
(1)214 cm^2. (2) 216 cm^2
(3) 218 cm^2 (4) 220 cm^2

22. A sphere of radius 8 cm is melted and recast into small spheres of radius 2 cm each. How many such small spheres are possible ?
(1)32 (2)64
(3) 16 (4) 8

23. The radius of the base of a cylinder is 7m and its height is 15 m. Its total surface area will be
(1) 938 sq.m. (2) 900 sq.m.
(3) 954 sq.m. (4) 968 sq.m.

24. The volume of a cone whose radius of the base is 3 cm and height 7 cm, is
(1) 62 cm^3 (2) 66 cm^3
(3) 68 cm^3 (4) 72 cm^3

25. The radius of the base of a cone is 7 cm and its slant height is 10 cm. The curved surface area of the cone is
(1)220 cm^2 (2) 210 cm^2
(3) 225 cm^2 (4) 240 cm^2

ANSWERS

I. 600 cm^2 **2.** 17 m
3. 640 cm^2 **4.** 8 cm
5. 1.5 cm **6.** 8m, 12m, 16 cm
7. 5120 cm^3 **8.** 2.1 m, 39.6 m^2
9. 4 m. **10.** 7 cm.
II. 10277.996 gms **12.** 198 cu. cm., 352 cu. m.
13. 4.235 cm. **14.** 46.2 m.
15.450 **16.** 4 cu. m.
16. (2) 16 m **17.** (3) Rs. 63.36
18. (1) 50 paise **19.** (2) 2 : 3
20. (3) $30\sqrt{2}$ m **21.** (2) 216 cm^2
22. (2) 64 **23.** (4) 968 sq.m.
24. (2) 66 cm^3 **25.** (1) 220 cm^2