RATIO & PROPORTION

Import ant Points :

Ratio : The ratio of two quantities of the same units is a fraction that one quantity is of the other.

The ratio $a : b$ represents a fraction $\frac{a}{b}$. The first term of a ratio is called antecedent while the second term is known as consequent. Thus, the ratio 5 : 7 represents $\frac{5}{7}$ with antecedent 5 änd consequent 7.

The multiplication or division of each term of a ratio by a same non-zero number does not affect the ratio.

Thus, $4 : 5 = 8 : 10 = 12 : 15 = 16 : 20 = \frac{4}{5} : 1$ etc.

Proportion : The equality of twojratios is called Proportion.

If $a : b = c$: d, we write, $a:b:: c:d$ and we say that a, b, c, d are in proportion.

In a proportion, the first and fourth terms are known as **extremes,** while second and third terms are known as **means.**

We have. **Product of Means • Product of Extremes**

Fourth Proportional: If $a: b$ = c: d, then d is called the fourth proportional to a, b, c.

Third Proportional: The third proportional to a, b is the fourth proportional to a, b, b.

Mean Proportional : Mean proportional between a and b is $\sqrt{ab}$.

Comparison of Ratios : We say that (a: b) > $(c : d)$ if $\frac{a}{b} > \frac{c}{d}$

Compounded Ratio : The compounded ratio of the ratios $(a : b), (c : d), (e,j)$ is $(ace : bdfl.$

Some More Definitions:

(i) $a^2 : b^2$ is called the **duplicate ratio** of $a: b$.

(ii) $\sqrt{a} : \sqrt{b}$ is called the **sub-duplicate ratio** of $a : b$.

(iii) $a^3 : b^3$ is called the **triplicate ratio** of $a : b$.

(iv) $a^{\frac{1}{3}} : b^{\frac{1}{3}}$ is called the **sub-triplicate ratio** of $a : b$.

(v) If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ (Componendo & dividendo)

Variation : We say that x is directly proportional to y if x = ky for some constant k and we write, x y, Also, we say that x is inversely proportional to y,

if $x = \frac{k}{y}$ for some constant k and we write, $x \propto \frac{1}{y}$.

SOLVED EXAMPLES

1. Cost of diamonmond is d varies directly as the Square of its weight. A diabroken into 4 pieces with their weights in the ratio 1 : 2 : 3 : 4. What is the price **of** the original diamond if the loss in total value of the diamond amounts to Rs. 140,000 ?

(SSC Graduate Level (Assistant Grade) Main Exam., 2000}

Sol. : Let the weights of 4 pieces of diamond be x Ix, 3x and 4x as the weights of pieces are in the ratio 1 : 2 : 3 : 4 .

∴ Total weight of the original Single diamond

= x + 2x + 3x + 4x = lOx

As given, cost of the diamond varies directly as [Square of its weight.

∴ Cost of the original diamond oe $(lOx)^2$

= $MIOOx^2$) = $100/cx^2$

Where, Kis proportionality constant

Total cost of the 4 pieces of diamond

= k $[Kx)^2 + (2x)^2 + (3x)^2 + (4x)^2$]

= k $[x^2 + 4X^2 + 9X^2 + 16X^2]$

= $k\ (30X^2) = 30\ kx^2$

Loss in total value of the diamond = 100 kx^2 - $30/a^2$

= $70/cx^2$

But this loss is equal to Rs. 140000

∴ $70kx^2$ = Rs. 140000

⇒ kx^2 = Rs. 2000 ..(i)

∴ Price of the original diamond = $100kx^2$

= Rs. (100 × 2000)

[From equation (i)

= Rs. 200,000

2. A contractor undertakes a contract of building toi prepare it in 8 months. He plans to complete the; work on hiring 210 workers working 20 daysa!

month and 6 hours a day. But he gets only 140 workers and he wants to eomplete the work in 12 months. How many hours would they have to work a day to eomplete the work in stipulated time?

(SSC Assistant Grade Main Exam. 1998)

Sol.: Here we are given 4 quantities and we have to find no. of working hours.

So. we compare each item to hours.

Less workers. more hours per day

(Indirect proportion)

More days, less hours per day

(Indirect proportion)

More months, less hours per day

(Indirect proportion)

Let the required hours be ***x.***

Workers	Months	Days	Hours/day
210 ↑	8 ↑	20 ↑	6
140	12	24	x ↓

$$\text{Therefore, } \left.\begin{matrix} 140 : 210 \\ 12 : 8 \\ 24 : 20 \end{matrix}\right\} :: 6 : x$$

$\Rightarrow 140 \times 12 \times 24 : 210 \times 8 \times 20 :: 6 : x$

$\Rightarrow 140 \times 12 \times 24 \times x = 210 \times 8 \times 20 \times 6$

$$\Rightarrow x = \frac{210 \times 8 \times 20 \times 6}{140 \times 12 \times 24} = 5 \text{ hours}$$

$\therefore$ The required number of hours = 5

3. In a factory the ratio of skilled, unskilled workmen and Clerks is 8 : 5 : 1. The ratio of their daily wages is 5 : 2 : 3 respectively. If on employing 20 unskilled workers, the total daily wages of all the workmen is Rs. 318, then find the daily wages of workers of each class.

(SSC Assistant Grade Main Exam, 1998)

Sol. : As given,

The ratio of the number of skilled, unskilled workmen and Clerks = 8 : 5 : 1

Number of unskilled workmen = 20

$$\therefore \text{ Number of skilled workmen } = \frac{8}{5} \times 20 = 32$$

$$\text{and number of clerks } = \frac{1}{5} \times 20 = 4$$

Ratio among their daily wages = 5 : 2 : 3

$\therefore$ The ratio of daily wages of the respective employees $= 5 \times 32 : 2 \times 20 : 3 \times 4$

$= 160 : 40 : 12 = 40 : 10 : 3$

Sum of the ratios $= 40 + 10 + 3 = 53$

$\therefore$ Daily wages of 32 skilled workmen

$$= \text{Rs. } \frac{40}{53} \times 318 = \text{Rs. } 240$$

Daily wages of 20 unskilled workmen

$$= \text{Rs. } \frac{10 \times 318}{53} = \text{Rs. } 60$$

$$\text{Daily wages of 4 clerks } = \text{Rs. } 3 \times \frac{318}{53} = \text{Rs. } 18$$

4. A bag contains Rs. 187 in the form of 1 rupe; 50 paise and 10 paise coins in the ratio 3 : 4 : 5. Find the number of each type of coins.

(SSC Graduate level Assistant Grade Main Exam, 2016)

Sol. : Ratio of 1 rupee, 50 paise and 10 paise coins = 3 : 4 : 5

Ratio of values of 1 rupee, 50 paise and 10 paise coins $= \frac{3}{1} : \frac{4}{2} : \frac{5}{10} = \frac{3}{1} : 2 : \frac{1}{2}$

$= 6 : 4 : 1$

Sum of the ratios $= 6 + 4 + 1 = 11$

$$\text{Value of 1 rupee coins } = \frac{6}{11} \times 187 = \text{Rs. } 102$$

$$\text{Value of 50 paise coins } = \frac{4}{11} \times 187 = \text{Rs. } 68$$

$$\text{Value of 10 paise coins } = \frac{1}{11} \times 187 = \text{Rs. } 17$$

$\therefore$ Number of 1 rupee coins = 102 x 1 = 102

Number of 50 paise coins = 68 x 2 = 136

Number of 10 paise coins = 17x10 = 170

5. The salaries of A, B and C are in the ratio of 1:2:3. The salary of B and C together is Rs. 6000. By what per cent is the salary of C more than that of A?

(1) 100% (2) 200%

(3) 300% (4) 600 %

Sol. Let the salaries of A, B, C be Rs. ***x, 2x*** and 3x respectively.

Then, $2x + 3x = 6000 \Rightarrow x = 1200$.

$\therefore$ As salary = Rs. 1200. Bs salary = Rs. 2400, and C's salary = Rs. 3600.

$$\therefore \text{ Excess of C's salary over A's } = \left(\frac{2400}{1200} \times 100\right)\%$$

$= 200\%$

Hence, option (2) is correct.

6. The average age of three boys is 25 years and their ages are in the proportion 3 : 5 : 7. The age of the youngest boy is :

(1) 21 years (2) 18 years

(3) 15 years (4) 9 years

Sol. : Total age of 3 boys = (25 × 3) years

= 75 years

$$\therefore \text{ Age of the youngest } = \left(75 \times \frac{3}{15}\right) = 15 \text{ years}$$

Hence, option (3) is correct.

7. An amount of Rs. 2430 ls divided among A, B and C such that if their shares be reduced by Rs. 5, Rs. 10 and Rs. 15 respectively. the remainders shall be in the ratio 3 : 4 : 5 . The share of B is :
(1) Rs. 605 (2) Rs. 790
(3) Rs. 800 (4) Rs. 810

Sol. : Remainder = Rs. |2430 - (5 + 10 + 15)1 • = Rs. 2400

$$\therefore \text{ B's share} = \text{Rs. } \left[\left(2400 \times \frac{4}{12}\right) + 10\right] = \text{Rs. } 810$$

Hence, Option (4) is correct.

8. The number of studenis in 3 classes are in the ratio 2 : 3 : 4. If 12 students are increased in each class, this ratio changes to 8 : 11 :14. The total number of students in the three classes in the beginning was
(1) 162 (2) 108
(3) 96 (4) 54

[SSS Graduate Level Prelim. Exam; 27.07.2008 (Ist Sitting)]

Sol. (1) Let the original number of students be 2x, 3x and 4x in three class.
According to the question.

$$\frac{2x+12}{3x+12} = \frac{8}{11}$$

$$\Rightarrow 24x + 96 = 22x + 132$$

$$\Rightarrow 2x = 132 - 96 = 36 \Rightarrow x = \frac{36}{2} = 18$$

∴ Original number of students
= 2x + 3x + 4x = 9x = 9 x 18 =162

9. A box contains 1 rupee, 50-paise and 25-paise coins in the ratio 8 : 5 : 3 . If the total amount of money in the box is Rs. 112.50, the number of 50-paise coins is
(1)80 (2)50
(3) 30 (4) 42

[SSS Graduate Level Prelim. Exam; 04.02.2007 (Ist Sitting)]

Sol. (2) Ratio of number of 1 rupee, 50-paise and 25 paise coins = 8 : 5 : 3
Ratio of their respective values

$$= 8 : \frac{5}{2} : \frac{3}{4} = 32 : 10 : 3$$

Sum of the ratios = 32 + 10 + 3 = 45
Value of 50 paise coins

$$= \text{Rs. } \left(\frac{10}{45} \times 112.5\right) = \text{Rs. } 25$$

∴ Number of 50 paise coins = 25x2 = 50

10. The ratio between two numbers is 3 : 4. If each number is increased by 6, the ratio becomes 4 : 5. The difference between the numbers is
(1) 1 (2) 3
(3) 6 (4) 8

[SSC CPO Sub-Inspector Exam; 06.09.2009]

Sol. (3) Let the numbers be 3x and 4x.

$$\therefore \frac{3x+6}{4x+6} = \frac{4}{5}$$

$\Rightarrow$ 16x + 24 = 15x + 30 $\Rightarrow$ x = 30 - 24 = 6
∴ Required difference = 6

11. If A: B = 3 : 2 and B : C = 3 : 4 then A: C is equal to
(1) 1 : 2 (2) 2 : 1
(3) 8 : 9 (4) 9 : 8

[SSC CPO Sub-Inspector Exam; 09.11.2008]

Sol. (4) A : B = 3 : 2
B : C = 3 : 4
A : B : C = 3 x 3 : 2 x 3 : 2 x 4 = 9 : 6 : 8

∴ A : C = 9 : 8 **OR.** $\frac{A}{B} \times \frac{B}{C} = \frac{3}{2} \times \frac{3}{4} = \frac{9}{8}$

12. In a cricket match to total number of runs scored by Sachin, Vinod and Sourav is 285. The ratio of the number of runs scored by Sachin and Sourav is 3 : 2 and that of the runs scored by Sourav and Vinod is also 3 : 2 . The number of runs scored by Sachin in that match is
(1) 135 (2) 90
(3) 60 (4) 140

(SSC Section Officer (Commercial Audit) Exam; 30.09.2007]

Sol. (1) Sachin : Sourav = 3 : 2
Sourav : Vinod = 3 : 2
Ratio of the runs scored by Sachin, Sourav and Vinod respectively = 3 x 3 : 2 x 3 : 2 x 2
= 9 : 6 : 4

$$\therefore \text{ Runs scored by Sachin} = \frac{9}{19} \times 285 = 135$$

13. In a bag, there are three types of coins — 1-rupee, 50 paise and 25-paise in the ratio of 3 : 8 : 20. Their total value is Rs. 372. The total number of coins is
(1) 1200 (2) 961
(3) 744 (4) 612

[SSC Section Officer (Commercial Audit) Exam, 30.09.2007]

Sol. (2) Ratio of the number of coins of Re. 1, 50 paise and 25 paise = 3 : 8 : 20
Ratio of the values of these coins

$$= 3 : \frac{8}{2} : \frac{20}{4} = 3 : 4 : 5$$

$$\text{Value of 1 rupee coins} = \frac{3}{12} \times 372 = \text{Rs. } 93$$

$$\text{Value of 50 paise coins} = \frac{4}{12} \times 372 = \text{Rs. } 124$$

$$\text{Value of 25 paise coins} = \frac{5}{12} \times 372 = \text{Rs. } 155$$

∴ Number of coins = 93 + 124 x 2 + 155 x 4
= 93 + 248 + 620 = 961

EVALUATE YOURSELF

SUBJECTIVE TYPE QUESTIONS

1. A contractor employed 15 men, each working 8 hours a day to do a certain piece of work in 19 days. At the end of 10 days, the work had to be suspended for 3 days owing to an accident, in which 4 men were disabled. How many more men must be engaged to eomplete the work in the speeified time, all the men now working 9 hours a day?

2. If 8 men and 12 boys, can finish a piece of work in 12 days, in what time will 40 men and 45 boys finish another piece of work 3 times as great, supposing that 16 men can do as much work in 8 hours as 12 boys can do in 24 hours.

3. If 25 men and 10 boys do as much work in 6 days as 21 men and 30 boys can do in 5 days. Compare the rate of doing work of a man and a boy. Also find that how many boys will eomplete the same work in 4 days with 40 men?

4. A man reaches a destination in 100 days taking rest for 8 hours a day. If he increases his speed thrice and rests for half of the previous one, then in how many days will he reach the thrice distant place, than previous one?

3. If the cost of 3 doors and 5 Windows be Rs. 487 and that of 5 doors and 3 window^ be Rs. 561, then find the cost of a door and a window.

6. ***A*** and ***B*** are two alloys of gold and copper prepared by mixing metals in proportions 7 : 2 and 7 : 1 1 respectively. If equal quantities of alloys are melted to form a third alloy C, then find the proportion of gold and copper in C.

7. In what ratio must 25% alcohol be mixed with 60% alcohol to get a mixture of 40% alcohol strength.

8. Two numbers are such that the ratio between them is 3 : 5 but if each is increased by 10, the ratio between them becomes 5 : 7. Find the numbers.

9. A grocer bought wheat, barley and dried peas. The ratio of the quantity of wheat in kg to the quantity of barley bought is the same as the ratio of quantity of. barley to that of dried peas. If he bought 2 quintals of wheat and 50 kg. of dried peas, find the quantity in kg. of barley bought by the grocer.

10. A grey hound pursues a hare and takes 5 leaps for every 6 leaps of the hare; but 4 leaps of the hound are equal to 5 leaps of the hare. Compare the rates of the hound and the hare.

11. A Student bought books, note books and pencils from a stationer. If the ratio of the number of books to the number of note books is the same as the ratio of the number of note books to the number of pencils, find the number of note books if the books and pencils are 20 and 5 respectively.

12. A pumping set of 2.7 horse power can raise 765 litres of water from a well of certain depth in a certain time. What horse power is needed to raise 3060 litres of water in the same time and from the same depth?

13. In a factory the ratio of male workers to female workers was 5 : 3, If the number of female workers was less by 40, find the total number of workers in the factory.

14. Two numbers are in the ratio 3 : 7. If 6 be added to each of them, they are in the ratio 5 : 9, find the numbers.

15.« A man undertakes to do a piece of work in 150 days. He employs 200 men. He finds that only a quarter of work is done in 50 days. How many additional men should be employed so that work may be finished in time?

OBJECTIVE TYPE QUESTIONS

16. If Rs. 1540 be divided amongst A, B and C in such a way that the share of B is equal to $\frac{3}{11}$ of what A and C together reeeive. Then, B's share will be :

(1) Rs. 330 (2) Rs. 420
(3) Rs. 880 (4) Rs. 1210

17. Rs. 1870 are divided into three parts in such a way that half of the first part, one-third of the second part and one-sixth of the third part are equal. The third part is :

(1) Rs. 510 (2) Rs. 680
(3) Rs. 850 (4) Rs. 1020

18. Rs. 2040 are divided among A, B and C such that A gets $\frac{2}{3}$ if what B gets and B gets $\frac{1}{4}$ of what C gets. Then Bs share is :

(1) Rs. 180 (2) Rs. 240
(3) Rs. 360 (4) Rs. 480.

19. A sum of money is divided among A, Band C such that to each rupee A gets, B gets 65 paise and C gets 35 paise. If C's share is Rs. 28, the sum is :

(1) Rs. 120 (2) Rs. 140
. (3) Rs. 160 (4) Rs. 180

20. Rs. 730 were divided among A, B and C in such a way that if A gets Rs. 3, then B gets Rs. 4 and if B gets Rs. 3.50 then C gets Rs. 3. The share of B exceeds that of C by :

(1) Rs. 30 (2) Rs. 40
. (3) Rs. 70 (4) Rs. 210

21. Rs. 53 are divided among three persons A, B and C in such a way that A gets Rs. 7 more than what B gets and B gets Rs. 8 more than what C gets. The ratio of their shares is :

(1) 16 : 9 : 18 (2) 25 : 18 : 10
(3) 18 : 25 : 10 (4) 15 : 8 : 30

22. Rs. 120 are divided among A, B and C such that As share is Rs. 20 more than B's and Rs. 20 less than C's. What is B's share?
(1) Rs. 10 (2) Rs. 15
(3) Rs. 20 (4) Rs. 25

23. A sum of Rs. 1300 is divided among P, Q, R and S such that:

$$\frac{\text{P's share}}{\text{Q's share}} = \frac{\text{Q's share}}{\text{R's share}} = \frac{\text{R's share}}{\text{S's share}} = \frac{2}{3}$$

What is P's share?
(1) Rs. 320 (2) Rs. 240
(3) Rs. 160 (4) Rs. 140

24. Rs. 600 are divided among A, B and C so that Rs. 40 more than $\frac{2}{5}$ th of As share, Rs. 20 more than $\frac{2}{7}$ th of B's share and Rs. 10 more than $\frac{9}{17}$ th of C's share may all be equal. What is A's share?
(1) Rs. 150 (2) Rs. 170
(3) Rs. 200 (4) Rs. 280

25. Rs. 180 contained in a box consists of one rupee, 50-paise and 25-paise coins in the proportion of 2 : 3 : 4 . What is the number of 50-paise coins?
(1) 120 (2) 150
(3) 180 (4) 240

26. Two numbers are in the ratio 3 : 4 and the product of their L.C.M. and H.C.F. is 10800. The sum of the numbers is :
(1) 180 (2) 210
(3) 225 (4) 240

27. The ages of X and Y are in the ratio of 3 : 1. Fifteen years hence, the ratio will be 2 : 1. Their present ages (in years) are :
(1) 30, 10 (2) 45, 15
(3) 21. 7 (4) 60, 20

28. The ratio of the number of boys and girls in a school is 3 : 2. If 20% of the boys and 25% of the girls are scholarship holders, the percentage of the school students who are not scholarship holders, is :
(1) 56 (2) 70
(3) 78 (4) 80

29. Gold is 19 times as heavy as water and copper is 9 times as heavy as water. In what ratio should these be mixed to get an alloy 15 times as heavy as water?
(1) 1:1 (2) 2 : 3
(3) 1 : 2 (4) 3 : 2

30. 85 kg of a mixture contains milk and water in a ratio 27 : 7. How much more water is to be added to get a new mixture containing milk and water in the ratio 3 : 1 ?
(1)5 kg (2) 6.5 kg
(3) 7.25 kg (4) 8 kg

31. 15 litres of a mixture contains 20% alcohol and the rest water. If **3** litres of water be mixed in it, the percentage of alcohol in the new mixture will be:
(1) 15 (2) $16\frac{2}{3}$
(3) 17 (4) $18\frac{1}{2}$

32. 20 litres of a mixture contains milk and water in the ratio 5 : 3. If 4 litres of this mixture are replaced by 4 litres of milk, the ratio of milk to water in the new mixture will become :
(1) 2 : 1 (2) 7 : 3
(3) 8 : 3 (4) 4 : 3

33. A and B are two alloys of gold and copper prepared by mixing metals in the ratio 7 : 2 and 7 : 11 respectively. If equal quantities of the alloys are melted to form a third alloy C. the ratio of gold and copper in C will be :
(1) 5 : 9 * (2) 5 : 7
(3) 7 : 5 (4) 9 : 5

34. A mixture contains milk and water in the ration 5 : 1 . On adding 5 litres of water, the ratio of milk to water becomes 5 : 2 . The quantity of milk in the original mixture is:
(1) 16 litres (2) 25 litres
(3) 22.75 litres (4) 32.5 litres

35. Two equal glasses are respectively $\frac{1}{3}$ and $\frac{1}{4}$ full of milk. They are then filled with water and the content« mixed in a tumbler. The ratio.of milk and water in the tumbler is :
(1) 7 : 5 (2) 7 : 17
(3) 9 : 21 (4) 11 : 23

ANSWERS

1. 9 men		**2.** 6 days		
3. 10 boys		**4.** 80 days		
5. Rs. 84, Rs. 47		**6.** 7 : 5		
7. 2 : **3**		**8.** 15 and 25		
9. 100 kg		**10.** 25 : 24		
11. 10		**12.** 10.8 HP.		
13. 160		**14.** 9 and 21		
15. 100 men.				
16. (1)	**17.** (4).	**18. (3)**	**19. (3)**	**20.** (2)
21. (2)	**22. (3)**	**23. (3)**	**24.**(1)	**25.** (1)
26. (2)	**27.** (2)	**28. (3)**	**29.** (4)	**30.**(1)
31. (2)	**32.** (2)	**33. (3)**	**34.** (2)	**35.** (2)